Mathematics - II

2015

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

*Answer **Eight** questions, selecting at least one from each Group.*

Group - A

1. a) State and prove Euler's theorem on homogeneous function of two variables.

   b) If $y=\left(\sin^{-1}x\right)^2$, then prove that

   $$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$$

2. a) Expand $\log(1+\tan x)$ in ascending powers of x using Maclaurin's series.

   b) Evaluate $\lim_{x\to 0}(\sin x)^{\tan x}$

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3. a) For the Pedal curve $p=f(r)$, prove that

   $$\rho=r\frac{dr}{dp}.$$

   b) Show that the curve represented by $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ for different values of n, have a common tangent at the point (a,b). Find the equation of common tangent.

Group - B

4. a) Evaluate any two of the following :

   i) $\int\frac{x^2-1}{x^4+1}dx$

   ii) $\int\frac{\cos x}{2\sin x+3\cos x}dx$

   iii) $\int_0^{\pi/4}\log(1+\tan\theta)\,d\theta$

   b) Find ab-initio value of $\int_a^b x^2\,dx$.

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5. a) If $I_n = \int_0^{\pi/4} \tan x \, dx$, prove that

$$I_n + I_{n-2} = \frac{1}{n-1}.$$

b) Find the perimeter of the loop of the curve $9y^2 = (x-2)(x-5)^2$.

6. a) Prove that $\overline{|(n)} = \underline{|n-1}$.

b) Find the area of loops of the curve $y^2 = x(x-1)^2$.

Group - C

7. Solve any two of the following :

a) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

b) $ydx - xdy = \sqrt{x^2 - y^2}\,dx$

c) $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$

d) $x\frac{dy}{dx} + y^2 = y^2 \log x$

8. a) Find the orthogonal trajectories of the family of parabola $y^2 = 4ax$ for different values of a.

b) Solve any one ozf the following :

i) $y = px + p - p^2$

ii) $p^2 y + 2px = y$

9. Solve any two of the following :

a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$

b) $(D^2 + D + 1)y = \sin 2x$

c) $(D^2 - 2D + 1)y = xe^x$

d) $(D^3 + 1)y = x^3 + \sin x$

Group - D

10. a) Prove that $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.

b) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then prove that the vectors $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are also non-coplanar.

11. a) Define curl of a vector field. Give the physical significance of curl of a vector field.

    b) Evaluate $\frac{d}{dt}\left[\vec{r}\times\frac{d\vec{r}}{dt}\times\frac{d^2\vec{r}}{dt^2}\right]$.

12. a) Prove that $\nabla\cdot(\phi\vec{u}) = \phi\nabla\cdot\vec{u} + \vec{u}\cdot(\nabla\phi)$ where ϕ is scalar point function and $\vec{u}$ is vector point function.

    b) If $\phi = x^3 + y^3 + z^3 - 3xyz$, find curl (grad ϕ).

Group - E

13. a) Obtain the necessary and sufficient conditions for the equilibrium of a system of coplanar forces acting on a rigid body.

    b) Forces P, Q, R act along the lines $x=0$, $y=0$ and $x\cos\theta + y\sin\theta = p$, the axes being rectangular. Find the magnitude of resultant and equation of its line of action.

14. a) Which forces can be omitted in forming the equation of virtual work?

    b) Six equal rods AB, BC, CD, DE, EF and FA are each of weight w and are freely jointed their extremities so as to form a hexagon. The rod AB is fixed in a horizontal position and the middle points of AB and DE are jointed by a string. Prove that the tension in string is thrice the weight of each rod.

15. a) State and explain Hooke's law. Find the work done in extending a light elastic string to double its length.

    b) A particle rests in equilibrium under the attraction of two centres of force which attract directly as the distance, their attractions per unit mass at unit distance being μ and μ'. The particle is slightly displaced towards one of them; show that the time of small oscillation is $\frac{2\pi}{\sqrt{\mu+\mu'}}$.

16. a) Find the radial and transverse acceleration of a particle moving in a plane curve.

    b) An insect crawls at a constant rate u along the spoke of a cartwheel of radius a, the cart is moving with a constant velocity v. Find the acceleration along and perpendicular to the spoke.