Roll No. ..... **Total Pages : 03**

CMDQ/M-20 5532

LINEAR PROGRAMMING

MSM-414

Time : Three Hours] [Maximum Marks : 70

Note : Attempt *Five* questions in all, selecting at least *two* questions from each Section.

Section I

1. (a) Prove that a necessary and sufficient condition for the existence and non-degeneracy of all possible basic solutions of the system $Ax = b$ is the linear independence of every set of m columns from the augmented matrix $[A : b]$. **7**

(b) If a closed and strictly bounded convex set has a finite number of extreme points, then prove that any point in the set can be written as a convex combination of the extreme points. **7**

2. (a) Consider a Linear Programming Problem (LPP) :

Maximize $Z = 3x_1 + 2x_2 + x_3$

Subject to the constraints :

$$3x_1 + 2x_2 + 2x_3 = 8$$

$$3x_1 + 4x_2 + x_3 = 7,\ x_i \geq 0.$$

Move from one basic feasible solution $x_3 = 3$, $x_2 = 1$ to another basic feasible solution such that the value of the objective function improves. **7**

(b) Explain slack and surplus variables. Give an example of each. What is the significance of these variables ? **7**

3. (a) Solve the following LPP using simplex method : **10**

$$8x_1 + 3x_2 + 4x_3 + x_4 \leq 7$$
$$2x_1 + 6x_2 + x_3 + 5x_4 \leq 3$$
$$x_1 + 4x_2 + 5x_3 + 2x_4 \leq 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Maximize $Z = 3x_1 + 4x_2 + x_3 + 7x_4$.

(b) Explain briefly the conversion of a minimization problem to a maximization problem. **4**

4. What is phase I of the two-phase method for artificial variables ? Solve the LPP : **14**

Maximize $Z = x_1 + 3x_2 + 2x_3 + 5x_4 + x_5 + 6x_6$

Subject to constraints :

$$3x_1 + 4x_2 + 5x_3 + x_4 + x_5 + x_6 = 8$$
$$x_1 + 3x_2 + 2x_3 + 5x_4 + 2x_5 + x_6 = 3$$
$$2.5x_1 + 5x_2 + 4.5x_3 + 5.5x_4 + 2.5x_5 + 4x_6 = 7$$
$$x_1 \ (i = 1, 2, \ldots\ldots, 6) \geq 0.$$

Section II

5. (a) Explain dual simplex algorithm. **7**

(b) Give geometrical interpretation of the perturbation method. **7**

6. Compare simplex and revised simplex methods. Using revised simplex method, solve the following LPP : **14**

Maximize $Z = x + 2y$

Subject to the constraints :

$$4x + 3y \leq 6$$
$$x + 6y \leq 3$$
$$x, y \geq 0.$$

7. (a) State and prove strong duality theorem. **7**

(b) Find the dual of : **7**

$$\begin{pmatrix} 1 & 4 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \leq \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$$x, y \geq 0.$$

Maximize $Z = 2x + y$.

8. Explain primal dual algorithm. Construct an example to illustrate steps of solving a LPP with primal-dual algorithm. **14**