Roll No. ....

0726

M. A./M. Sc. (Previous)

EXAMINATION, 2022

MATHEMATICS

Paper Third

(Topology)

Time : Three Hours] [*Maximum Marks : 90*

Note : Attempt questions from all Sections as directed.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A

(Short Answer Type Questions)

Note : All questions are compulsory. Each question carries 3 marks.

1. (A) Let X be a metric space. Then any finite intersection of open sets in X is open.

   (B) In a metric space every convergent sequence is a Cauchy sequence.

   (C) Find all possible topologies for the set $X = \{1, 2, 3, 4\}$.

   (D) Show that every T_2-space is a T_1-space.

   (E) If (X, **T**) be a topological space, then prove that the identity mapping :

   $$f : X \rightarrow X : f(x) = x \quad \forall x \in X$$

   is continuous.

   (F) Continuous image of a connected space is connected.

   (G) Closed subsets of compact sets are compact.

(H) Every second countable space is first countable.

(I) The product space of Hausdorff space is Hausdorff.

(J) Define the following with example :

(i) Nets

(ii) Convergence of net

Section—B

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 15 marks.

2. Let X be a complete metric space and let Y be a subspace of X. Then Y is complete if and only if it is closed.
3. State and prove Baire category theorem.
4. State Zermelo's postulate and prove that Zermelo's postulate is equivalent to the axiom of choice.
5. Let X be a topological space. Then (i) any intersection of closed sets in X is closed and (ii) any finite union of closed sets in X is closed.

Section—C

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 15 marks.

6. Let (X, **T**) and (Y, V) be topological spaces and let f be a bijective mapping (i. e. one-one onto of X to Y. Then the following statements are equivalent :

   (i) f is a homeomorphism.

   (ii) f is continuous and open.

   (iii) f is continuous and closed.
7. A subset E of R is connected if and only if it is an interval in particular R is connected.
8. State and prove Lindelof's theorem.
9. A topological space (X, **T**) is Hausdorff if and only if every net in X can converges to at most one point.