2022/TDC(CBCS)/EVEN/SEM/
STSHCC–202T/124

TDC (CBCS) Even Semester Exam., 2022

STATISTICS

(Honours)

(2nd Semester)

Course No. : STSHCC–202T

(Algebra)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* questions : 2×10=20

1. If a, b, c are the roots of $x^3 + px^2 + qx + r$, then find the value of

$$\sum \frac{b^2 + c^2}{bc}$$

2. Determine a polynomial whose roots are –3, –1, 4, 5.

3. Find the condition that the cubic equation $x^3 - px^2 + qx - r = 0$ should have roots in GP.

4. Define vector space and basis of a vector space.

5. Define linear dependence and linear independence of vector.

6. Show that

$$S = \{(1, 2, 4), (1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

the set is linearly dependent sub-space of the vector space $V_3(R)$, where R is the field of real numbers.

7. Define idempotent matrix and nilpotent matrix.

8. Define trace of a matrix.

9. Define Hermitian and skew-Hermitian matrices.

10. Define minors and cofactors.

11. Define determinant of a square matrix.

12. Define inverse of a matrix.

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13. Define rank of a matrix.

14. Define echelon form of a matrix.

15. Define characteristic roots and characteristic vectors.

SECTION—B

Answer any *five* questions : 6×5=30

16. *(a)* Show that an algebraic equation of degree n has n roots. 3

(b) Solve by Cardan's method

$$x^3 + 6x^2 - 12x + 32 = 0$$ 3

17. If sum of two roots of $x^3 + px^2 + qx + r$ is equal to the third root, then show that $p^3 - 4pq + 8r = 0$.

18. *(a)* Show that the three vectors $(1, 1, -1)$, $(2, -3, 5)$ and $(-2, 1, 4)$ of R^3 are linearly independent. 2

(b) If

$\alpha_1 = (1, 2, -1)$, $\alpha_2 = (-3, -6, 3)$, $\alpha_3 = (2, 1, 3)$

then show whether the system is linearly dependent or independent. 4

19. *(a)* State the general properties of vector space. 2

(b) Determine whether or not the following vectors form a basis of R^3 : 4

$\alpha_1 = (1, 1, 2)$, $\alpha_2 = (1, 2, 5)$, $\alpha_3 = (5, 3, 4)$

20. *(a)* Let A and B be two square matrices of order n and λ be a scalar. Then prove that $\text{tr}(AB) = \text{tr}(BA)$. 2

(b) Prove that a necessary and sufficient condition for a matrix A to be symmetric is that A and A' are equal. 4

21. Show that every square matrix A can be uniquely expressed as $P + iQ$, where P and Q are Hermitian matrices.

22. *(a)* Show that

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$ 3

(b) Prove that

$$\Delta = \begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} = (x+3a)(x-a)^3$$ 3

23. *(a)* If A, B be two n-rowed non-singular matrices, then prove that AB is also non-singular and $(AB)^{-1} = B^{-1}A^{-1}$. 3

(b) Solve the following system of linear equations with the help of Cramer's rule : 3

$$x + 2y + 3z = 6$$
$$2x + 4y + z = 7$$
$$3x + 2y + 9z = 14$$

24. State and prove Cayley-Hamilton theorem.

25. *(a)* Show that the two matrices A, $C^{-1}AC$ have the same characteristic roots. 3

(b) Prove that if the characteristic roots of A are $\lambda_1, \lambda_2, \ldots, \lambda_n$, then the characteristic roots of A^2 are $\lambda_1^2, \lambda_2^2, \ldots, \lambda_n^2$. 3

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