MATHEMATICS Paper I: Advanced Calculus

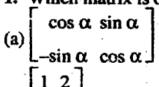
Tapet 1 . Advanced Calculus

Note: Attempt all questions from Section-A (Objective type), five questions from Section-B (Short answer type) and three questions from Section-C (Long/Essay type questions).

Section-A

 $1 \times 10 = 10$

1. Which matrix is orthogonal:



(b)
$$\begin{bmatrix} \sin \alpha & -\sin \alpha \\ \cos \alpha & \cos \alpha \end{bmatrix}$$

(c)
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(d) None of these.

2. What is order of resultant matrix

$$[xyz] \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

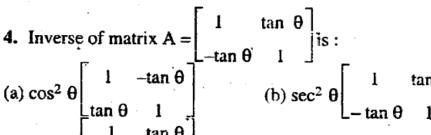
- (a) (3×1)
- (b) (1×3)
- (c) (1×1)
- (d) (3×3) .

UPadda.com

3. Which one is skew her nitian matrix:

(a)
$$\begin{bmatrix} 1 & -1+i \\ -1+i & 0 \end{bmatrix}$$
(c)
$$\begin{bmatrix} 0 & 1-i \\ 1+i & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 0 & -1+i \\ -1+i & 0 \end{bmatrix}$$
(d)
$$\begin{bmatrix} i & -1+i \\ 1 & i & 0 \end{bmatrix}$$



- (c) $\cos^2 \theta$ $\begin{bmatrix} 1 & \text{ta} \\ -\text{tan } \theta \end{bmatrix}$
- (d) None of these.
- 5. Rank of matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is:
- (a) 2
- (b) 3
- (c).1
- (d) None of these.

- 6. Which are true in following:
- (a) (C, + ·) is a vector space over field R
- (b) (R, +) is a vector space over field C
- (c) Q + ·) is a vector space over field R
- (d) Set of integers is vector space over field Q.

- 7. The vectors are linearly independent:
- (a) $\{(1, 2, 3), (1, 0, 0), (0, 1, 0), (001)\}$ (b) $\{(100), (010), (001)\}$
- (c) {(100), (010), (111), (001)}
- (d) {(123), (111), (100)}.
- 8. In following statements which one is true:
- (a) If A and B are subspaces of a vector space then $A \neq B \implies \dim A \neq A$ dim B (b) If A and B are subspaces of a vector space then dim A < dim B \Rightarrow A \subseteq B (c) If S₁ and S₂ are finite dim. subspaces with the same dim and if $S_1 \subseteq S_2$ then $S_1 = S_2$.
 - 9. Which of following mappings are linear transformations:
 - (a) $T: V_2(R) \to V(R): T(a, b) = a b$
 - (b) $T: V_3(R) \to V_2(R): T(a, b, c) = (a+1, b+c)$
 - (c) $T: V_2(R) \to V_2(R): T(a) = (2a, 3a)$.
 - 10. Which one is non-singular matrix:

(a)
$$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$
 (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} \sin \alpha & \cos \alpha \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$.

Section—B $2 \times 5 = 1$

Solve the equations by matrix method :

$$x + 2y + z = 2$$
, $2x + 6y + 5z = 4$, $2x + 4y + 3z = 3$.

2. Find the characteristic roots of matrix.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

- $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ 3. Find the rank of matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$.
- 4. Show that the sets of vectors (1, 0, 0) (1, 1, 0) (1, 1, 1) and (0, 1, 0)in V_3 R) is not basis set.
 - 5. If in an inner product space

$$||x+y|| = ||x|| + ||y||$$

then prove that, x, y are linearly dependent.

$$15 \times 3 = 15$$

- 1. To show that the there exists a basis for each finite dimensional vector space.
- 2. (a) Show that every square matrix satisfies its characteristic equation.
- (b) Define Rank and nullity of linear transformation and write the relation between them.
- 3. (a) A vector space V(F) to be a direct sum of its two subspaces w_1 and w_2 iff (i) $V = w_1 + w_2$, (ii) w_1 and w_2 are disjoint, $w_1 \cap w_2 = (0)$.
- (b) If A and B are two non-singular matrices of same order then AB is also non-singular and $(AB)^{-1} = B^{-1} \cdot A^{-1}$

- 4. (a) Find the rank of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 5 \\ -1 & 0 & -3 \end{bmatrix}$
- (b) Solve the equations by matrix method: x + 2y + z + 2, 2x + 6y + 5z = 4, 2x + 4y + 3z = 3.
- 5. (a) Find the characteristic roots of matrix and show that matrix satisfies its characteristic equation:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

(b) Find a non-singular matrix P such that D = P'AP.

where
$$A = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & -7 \end{bmatrix}$$
 and D is a diagonal matrix.