MATHEMATICS

Paper I : Advanced Calculus

Note : Attempt **all** questions from **Section-A** (Objective type), **five** questions from **Section-B** (Short answer type) and **three** questions from **Section-C** (Long/Essay type questions).

Section—A

$1 \times 10 = 10$

1. Which matrix is orthogonal :

(a) $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ (b) $\begin{bmatrix} \sin\alpha & -\sin\alpha \\ \cos\alpha & \cos\alpha \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) None of these.

2. What is order of resultant matrix :

$$[xyz]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(a) (3×1) (b) (1×3) (c) (1×1) (d) (3×3).

3. Which one is skew hermitian matrix :

(a) $\begin{bmatrix} 1 & -1+i \\ -1+i & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1+i \\ -1+i & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1-i \\ 1+i & 0 \end{bmatrix}$ (d) $\begin{bmatrix} i & -1+i \\ 1-i & 0 \end{bmatrix}$.

4. Inverse of matrix $A = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ is :

(a) $\cos^2\theta \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$ (b) $\sec^2\theta \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$

(c) $\cos^2\theta \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ (d) None of these.

5. Rank of matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is :

(a) 2 (b) 3 (c) 1 (d) None of these.

6. Which are true in following :

(a) (C, + ·) is a vector space over field R

(b) (R, + ·) is a vector space over field C

(c) Q + ·) is a vector space over field R

(d) Set of integers is vector space over field Q.

7. The vectors are linearly independent :

(a) {(1, 2, 3) (1, 0, 0) (0, 1, 0) (001)} (b) {(100), (010), (001)}

(c) {(100), (010), (111), (001)} (d) {(123), (111), (100)}.

8. In following statements which one is true :

(a) If A and B are subspaces of a vector space then $A \neq B \Rightarrow \dim A \neq \dim B$ (b) If A and B are subspaces of a vector space then $\dim A < \dim B \Rightarrow A \subseteq B$ (c) If S_1 and S_2 are finite dim. subspaces with the same dim and if $S_1 \subseteq S_2$ then $S_1 = S_2$.

9. Which of following mappings are linear transformations :

(a) $T : V_2(R) \rightarrow V(R) : T(a, b) = a - b$

(b) $T : V_3(R) \rightarrow V_2(R) : T(a, b, c) = (a + 1, b + c)$

(c) $T : V_2(R) \rightarrow V_2(R) : T(a) = (2a, 3a)$.

10. Which one is non-singular matrix :

(a) $\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} \sin\alpha & \cos\alpha \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$.

Section—B

$2 \times 5 = 10$

1. Solve the equations by matrix method :

$x + 2y + z = 2, 2x + 6y + 5z = 4, 2x + 4y + 3z = 3.$

2. Find the characteristic roots of matrix.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

3. Find the rank of matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$.

4. Show that the sets of vectors (1, 0, 0) (1, 1, 0) (1, 1, 1) and (0, 1, 0) in V_3 R) is not basis set.

5. If in an inner product space

$$\|x + y\| = \|x\| + \|y\|$$

then prove that, x, y are linearly dependent.

Section—C

$15 \times 3 = 15$

1. To show that the there exists a basis for each finite dimensional vector space.

2. (a) Show that every square matrix satisfies its characteristic equation.

(b) Define Rank and nullity of linear transformation and write the relation between them.

3. (a) A vector space V(F) to be a direct sum of its two subspaces w_1 and w_2 iff (i) $V = w_1 + w_2$, (ii) w_1 and w_2 are disjoint, $w_1 \cap w_2 = (0)$.

(b) If A and B are two non-singular matrices of same order then AB is also non-singular and $(AB)^{-1} = B^{-1}.A^{-i}$

4. (a) Find the rank of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 5 \\ -1 & 0 & -3 \end{bmatrix}$.

(b) Solve the equations by matrix method :

$x + 2y + z + 2, 2x + 6y + 5z = 4, 2x + 4y + 3z = 3.$

5. (a) Find the characteristic roots of matrix and show that matrix satisfies its characteristic equation :

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

(b) Find a non-singular matrix P such that D = P'AP.

where $A = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & -7 \end{bmatrix}$ and D is a diagonal matrix.