Paper II : Complex Analysis

M.M. B.Sc. : 50 / B.A. : 25

Section—A

B.Sc. : $1 \times 10 = 10$
B.A. : $0{\cdot}5 \times 10 = 5$

1. Find Moduli $z = (\sqrt{2} - i) - i(1 - i\sqrt{2})$.
2. The inverse of the point z w.r.t. the circle $|z| = 1$ is 1/2. True/False
3. If C is a circle $|z| = r > 0$, then $\int_C \frac{dz}{z^2}$ is equal to 0. True/False
4. The value of $\int_C \frac{dz}{z^2 - 1}$, where C is $x^2 + y^2 = 4$ is 0. True/False
5. The transformation $w = \left(\frac{1 + z^n}{1 - z^n}\right)^2$ maps any circular sector on the semi-plane. True/False
6. Find the fixed point of $w = \frac{z}{z - 2}$.
7. $z = 0$ is an isolated essential singularity of the function $e^{1/z}$. True/False
8. The function $f(z) = \sin(z)$ is bounded. True/False
9. If $f(z) = \frac{z^2}{(z-1)(z-2)(z-3)}$, then residue of $f(z)$ at $z = 1$ is 1/2. True/False
10. Define entire function.

Section—B

B.Sc. : $2 \times 5 = 10$
B.A. : $1 \times 5 = 5$

1. Find the locus of the point z for given condition : $avg \frac{z-1}{z+1} = \frac{\pi}{6}$.
2. Find the radius of convergence of the series :
$$\frac{1}{2}z + \frac{1.3}{2.5}z^2 + \frac{1.3.5}{2.5.8}z^3 + \ldots$$
3. Discuss fully the transformation $w = \cos hz$.
4. Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along the path $y = x$.
5. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in the region $|z| < 1$.
6. Find the number of isolated singular points of $f(z) = \frac{z+3}{z^2(z^2+2)}$.
7. Evaluate $\int_C \frac{2z^2 + 5}{(z+2)^3(z^2+4)} dz$, where C is the square with the vertices at $1 + i$, $2 + i$, $2 + 2i$ and $1 + 2i$.
8. Evaluate the residue of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at $z = 2$.

Section—C

B.Sc. : $10 \times 3 = 30$

B.A. : $5 \times 3 = 15$

1. (a) Prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \, \partial z}$.

(b) If $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z.

2. (a) Find the image of $|z - 3i| = 3$ under the mapping $w = 1/z$.

(b) Show that the transformation $w = \frac{2}{\sqrt{2z}} - 1$ transformation the region outside the parabola $y^2 = 4(1 - x)$ into the interior of the unit circle in w-plane.

3. State and prove Liouville's Theorem.

4. State and prove Rouche's Theorem.

5. Using contour integration, evaluate :

$$\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} \, d\theta.$$