Paper II : Abstract Algebra

Section—A $0{\cdot}5 \times 10 = 5; 0{\cdot}8 \times 10 = 8$

1. How many times $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$ should be multiplied by itself to get $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$:

(a) 2 (b) 3 (c) 4 (d) None.

2. (R, +, ·) is a ring of real nos and (Q, +, ·) is sub ring of rational nos then (Q, +, ·) is an ideal. True/False

3. The polynomial over the ring of $(I_8, +_8, \times_8)$ are , $f(x) = 2 + 6x + 4x^2$, $g(x) = 2x + 4x^2$. Then degree of $[f(x) + g(x)]$ is :

(a) 2 (b) 1 (c) 0 (d) None of these.

4. V_3 (R) = {(a, b, c) : a, b, c ∈ R} be a vector space and W_1 = {a, 0, 0} : a ∈ R}, W_2 = {0, b, 0) : b ∈ R} be sub-spaces of V_3 (R) then $W_1 \cup W_2$ is sub subspace of v_3 (R). True/False

5. Permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 1 & 4 & 8 & 6 & 9 & 7 & 5 \end{pmatrix}$ = product of permutations = (4) (6) (123) (5879) True/False

6. Subset S = {(100), (110), (111), (010)} of R^3 (R) is basis set. True/False

7. Any orthogonal set of non-zero vectors in an inner product space V is.........

8. If T be linear transformation from vector space u(F) into vector space v (F) with u is finite dimensional. Then (rank T) + (nullity T) = ?

9. If F is a field then the set F(x) of all polynomials over F is (a field/an integral domain).

10. If V(F) be an inner product space and two of its vectors x, y are such that $|(x, y)| = \|x\| \cdot \|y\|$. Then x, y are linearly dependent/linearly independent.

Section—B $1 \times 5 = 5; 2 \times 5 = 10$

1. If α and β are vectors in a real inner product space and if $\|\alpha\| = \|\beta\|$ then (α – β) and (α + β) are orthogonal.

2. Show that the mapping T : V_3 (R) → V_2 (R) defined as a linear transformation from V_3 (R) into V_2 (R).

$$T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3, a_1 - 3a_2 - 2a_3).$$

3. Define normal subgroup. If H is sub group of index 2 in a group G, then H is a normal subgroup of G.

4. Let V be the vector space C(R), show that the set S = [(1, 0), (0, 1), (i, 0), (0, i)] is a basis for V.

5. Prove the necessary conditions for a vector space V (F) to be direct sum of its two subspaces W_1 and W_2 are that : (i) $V = W_1 + W_2$ and (ii) $W_1 \cap W_2 = (0)$.

Section—C $\quad 5 \times 3 = 15$

1. Every finite-dimensional inner product space has an orthonormal basis.

2. (a) If W_1 and W_2 are two subspaces of a finite dimensional vector space V(F), then $\dim (W_1 + W_2) = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2)$.

(b) Show that mapping $T : V_2(R) \rightarrow V_3(R)$ given by $T(a, b) = [a + b, a - b, b)$ is a linear Transform from $V_2(R)$ into $V_3(R)$.

3. (a) Prove that : Every finite group G is isomorphic to a permutation group.

(b) If V(F) is a finite dimensional vector space, then any two bases of V have same nos of elements.

4. Prove that :

(a) The centre Z of a group G is a normal subgroup of G.

(ii) Prove that ring of integers is an Euclidean ring.

5. (a) Show that vectors (1, 2; 1) (2, 1, 0) (1, – 1, 2) form a basis of R^3.

(b) Prove that in $|\underline{n}$ permutation on n symbols, $\frac{|\underline{n}}{2}$ are even permutation and $\frac{|\underline{n}}{2}$ are odd permutation.