

9. Find the particular integral of $(D - a)^2 y = e^{ax} \sin x$.

10. Solve the equation $x^2 y'' - xy' + y = 0$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find $\frac{dy}{dx}$ if $y = x^2 e^{2x} (x^2 + 1)^4$. (8)

(ii) For what value of the constant b , is the function f continuous on $(-\infty, \infty)$ if $f(x) = \begin{cases} bx^2 + 2x & \text{if } x < 2 \\ x^3 - bx & \text{if } x \geq 2. \end{cases}$ (8)

Or

(b) If $f(x) = 2x^3 + 3x^2 - 36x$, find the intervals on which it is increasing or decreasing, the local maximum and local minimum values of $f(x)$. (16)

12. (a) (i) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, then find $\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z}$. (8)

(ii) Find the shortest and the longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$. (8)

Or

(b) (i) Expand $x^2 y^2 + 2x^2 y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$ using Taylor's series upto third degree terms. (8)

(ii) Examine $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values. (8)

13. (a) (i) Evaluate $\int_0^{\infty} e^{-ax} \sin bx \, dx$ ($a > 0$) using integration by parts. (8)

(ii) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\cos^2 x + 3 \cos x + 2} \, dx$. (8)

Or

(b) (i) Evaluate $\int \frac{2x+5}{\sqrt{x^2-2x+10}} \, dx$. (8)

(ii) Evaluate $\int_0^{\frac{\pi}{4}} x \tan^2 x \, dx$. (8)

14. (a) (i) Change the order of integration in $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} \, dy \, dx$ and then evaluate it. (8)

(ii) Evaluate, by changing to polar coordinates $\int_0^a \int_y^a \frac{x^2 \, dx \, dy}{\sqrt{x^2 + y^2}}$. (8)

Or

(b) (i) Evaluate $\iint xy \, dx \, dy$ over the region in the positive quadrant bounded by $\frac{x}{a} + \frac{y}{b} = 1$. (8)

(ii) Find the value of $\iiint xyz \, dz \, dy \, dx$ through the positive spherical octant for which $x^2 + y^2 + z^2 \leq a^2$. (8)

15. (a) (i) Solve by the method of variation of parameters :

$$\frac{d^2 y}{dx^2} + a^2 y = \tan ax. \quad (8)$$

(ii) Solve $(D^2 + 2D + 1)y = e^x \sin 2x$ by using the method of undetermined coefficients. (8)

Or

(b) (i) Solve $(x+2)^2 \frac{d^2 y}{dx^2} - (x+2) \frac{dy}{dx} + y = 3x + 4$. (8)

(ii) Solve $\frac{dx}{dt} + \frac{dy}{dt} + 3x = \sin t, \frac{dx}{dt} + y - x = \cos t$. (8)