22/12/14 – FN

Reg. No. :

Question Paper Code : 97100

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

First Semester

Mechanical Engineering

MA 6151 — MATHEMATICS — I

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If 2, 3 are the eigenvalues of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$, then find the value of b.

2. State Cayley-Hamilton theorem.

3. Using integral test, determine the convergence of $1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n-1}+\ldots$

4. Using comparison test, prove that the series $\frac{1}{1.3}+\frac{2}{3.5}+\frac{3}{5.7}+\ldots$ is divergent.

5. Find the radius of curvature of the curve $y=e^x$ at $(0, 1)$.

6. Define involutes and evolutes.

7. If $u=\frac{y}{z}+\frac{z}{x}+\frac{x}{y}$, then find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$.

8. Find the Jacobian of the transformation $x=r\cos\theta$, $y=r\sin\theta$.

9. Evaluate $\int_2^3\int_1^2 \frac{1}{xy}\,dx\,dy$.

10. Evaluate $\int_0^{\pi/2}\int_0^{\sin\theta} r\,d\theta\,dr$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Using Cayley-Hamilton theorem find A^4 for the matrix $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. (8)

(ii) Find the eigenvalues and eigenvectors of $\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$. (8)

Or

(b) (i) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form through orthogonal transformation. (10)

(ii) If β is an eigenvalue of a matrix, then prove that $\frac{1}{\beta}$ is the eigenvalue of A^{-1}. (6)

12. (a) (i) Show by direct summation of n terms that the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$ is convergent. (8)

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1} x^n, \ x > 0$. (8)

Or

(b) (i) Determine convergence of an alternating series and test $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2+1}$ for absolute and conditional convergence. (8)

(ii) Test the convergence of the series $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \ldots$ by D'Alembert's ratio test. (8)

13. (a) (i) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are connected by $a^2 + b^2 = 64$. (8)

(ii) Find the evolute of the cycloid $x = a(\theta - \sin\theta), \ y = a(1 - \cos\theta)$. (8)

Or

(b) (i) Find the equation of the circle of curvature of $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$. (8)

(ii) Find the radius of curvature at the point $(a,0)$ on the curve $xy^2=a^3-x^3$. (8)

14. (a) (i) Find the extreme value of the function $f(x, y)=x^3+y^3-3x-12y+20$. (8)

(ii) If $u=(x-y)f\left(\frac{y}{x}\right)$, then find $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$. (8)

Or

(b) (i) Find the length of the shortest line from the point $\left(0, 0, \frac{25}{9}\right)$ to the surface $z=xy$. (8)

(ii) Expand $\sin xy$ at $\left(1,\frac{\pi}{2}\right)$ upto second degree terms using Taylor's series. (8)

15. (a) (i) Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (8)

(ii) Find the area of the cardioid $r=a\,(1+\cos\theta)$. (8)

Or

(b) (i) Change the order of integration $\int_0^\infty\int_0^y ye^{-y^2/x}\,dx\,dy$ and hence evaluate it. (8)

(ii) Evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}}(x^2+y^2)\,dy\,dx$ by changing into polar co-ordinates. (8)

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