B.SC (H) PART I

D.P. - I, EXAMINATION - 2021

MATHEMATICS (HONS.)

PAPER - I

Time : 3 Hours **Maximum Marks : 100**

Note: Candidate are required to give their answer in their own words as far as practicable. Question No. 1 is compulsory and answer five more questions. Selecting at least one from each group. Questions number 2 to 13 are of equal marks.

1. Answer any ten of the following: 10x2=20

(i) If R and S are two relations and R = $\{(2,3)\}$ S = $\{(3,4)\}$ Then composition of R and S i.e. ROS given by

(a) (2,4) (b) (2,3)

(c) (4,2) (d) None

(ii) If $x = \{1,2\}$, then $(P(x), \subseteq)$ is a

(a) Partially ordered as well as totally ordered

(b) Not partially ordered set but totally ordered

(c) Partially ordered set but not totally ordered set

(d) None

(iii) For any non-empty set A

(a) Card A $\leq$ Card P (A) (b) Card A $\geq$ Card P (A)

(c) Card A $<$ Card P (A) (d) Card A $>$ Card P (A)

(iv) The diagonal element of a skew-Hermitian matrix is

(a) Zero

(b) Pure imaginary

(c) Either zero or pure imaginary

(d) Either zero or purely real.

(v) If det $(A^{-1}) = (\det A)^{-1}$. Then matrix A is

(a) Singular

(b) Non-singular

(c) Either singular or non-singular

(d) None

(vi) A system of equation AX=B is non-homogenous system of equation if B is

(a) Null Matrix (b) Not Null matrix

(c) Either null or not null matrix (d) None

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(vii) If $|X|<1$ then $1+\frac{1}{2}x+\frac{1}{2}\cdot\frac{3}{4}x^2+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}x^3+$ .... is equal to

(a) $(1+x)^{-\frac{1}{2}}$ (b) $(1-x)^{-\frac{1}{2}}$

(c) $(1+x)^{\frac{1}{2}}$ (d) None

(viii) If a,b,c are positive and not all are equal, then (a+b+c) (ab+bc+ca) is

(a) Greater than 9 a b c 12

(b) Greater than and equal to 9 a b c

(c) Less than 9 a b c

(d) None

(ix) In continued fraction $\frac{1}{1+}\frac{1}{2+}\frac{1}{3+}\frac{1}{4+}\frac{1}{3+}\frac{1}{4+}$...........

(a) $\frac{1}{1+}\frac{1}{2}$ is acyclic part and $\frac{1}{3+}\frac{1}{4+}$ is cyclic part

(b) $\frac{1}{3+}\frac{1}{4+}$ is acyclic part and $\frac{1}{1+}+\frac{1}{2}$ is cyclic part

(c) $\frac{1}{1+}\frac{1}{2}$ and $\frac{1}{3+}\frac{1}{4+}$ are both cyclic part

(d) None

P.T.O.

(x) Every equation of odd degree has at least one is

(a) Real root (b) Complex Rot

(c) Real or Complex root (d) None

(xi) If $f(x)$ be a polynomial and a, b are reals such that f(a) and f(b) are of opposite signs, then

(a) At least one or an odd number of real roots of $f(x)=0$ lie between a and b

(b) At least one or even number of real roots of $f(x)=0$ lie between a and b

(c) Only one real root of $f(x)=0$ lie between a and b

(d) None

(xii) If α,β,Γ be the roots of equation $x^3+px^2+qx+r=0$ then $\sum\alpha^2$ is equal to

(a) P^2-2q (b) P^2+2q

(c) $2q-p^2$ (d) P^3-2q

GROUP -A

2. State and prove fundamenal theorem of an equivalence relation. 16

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3. (a) Distinguish between partially ordered set and totally ordered set and give an example of a partially ordered set which is not totally ordered set.

(b) Prove that $(R, \le)$ is a partially ordered set which is also a totally ordered with $\le$ is usual meaning.

4. (a) Prove that set of real numbers R is uncountable.

(b) If E is any set then prove that card $P(E) = 2^{\text{card}\,E}$, where P(E) is the power set of E.

GROUP -B

5. (a) Define unitary matrix and prove that

$$A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$$ is unitary.

(b) Show that every square matrix is uniquely expressible as the sum of a Hermitian matrix and a skew-Hermitian matrix.

6. (a) Define inverse of a matrix and prove that the necessary and sufficient condition for the existance of the inverse of a square matrix A is that A is non-singular.

(b) Define adjoint of a matrix and prove that

A.(adj A) = (adj A).A = /A/.I

7. Show that the equations

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

are consistent and solve them.

GROUP C

8. (a) Which is the numerically greatest term (or terms) in expansion of $(1+x)^{25/2}$ when $x = \frac{4}{5}$?

(b) Sum of the series $1 + \frac{5}{8} + \frac{5.8}{8.12} + \frac{5.8.11}{8.12.16} + \ldots\ldots\forall$

9. (a) Resolve into the simplest possible partial fractions.

(b) Prove that $\frac{1}{2\sqrt{n+1}} < \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdots\frac{2n-1}{2n} < \frac{1}{\sqrt{(2n+1)}}$

10. (a) Prove that any rational number can be expressed as a simple terminating continued fraction which can be arranged so as to have either an odd or an even number of quotients.

(b) Use the H.C.F process to express $(1-x)^{-2}$ as a continued fraction.

GROUP - D

11. (a) Prove that every equation of nth degree has n roots and no more.

(b) Find the condition that the equation

$x^4 + px^3 + qx^2 + rx + s = 0$

Should have two roots connected by the relation $\propto + \beta = 0$, Determining in that case quadratic equations which shall have for roote

(i) $\propto \beta$ and (ii) $\Gamma \delta$

12. (a) If $\propto, \beta, \Gamma$ are roots of $x^3 - px + q = 0$ prove that $35_2 5_5 = 55_3 5_4$

(b) If $\propto, \beta, \Gamma$ be the roots of equation $x^3 - 2x - 3 = 0$, find the equation whose roots are $\frac{1+\propto}{1-\propto}, \frac{1+\beta}{1-\beta}, \frac{1+\Gamma}{1-\Gamma}$

13. (a) If $\propto, \beta, \Gamma$ be the roots of the equation $x^3 + 3x + 9 = 0$, find the value of $\propto^9 + \beta^9 + \Gamma^9$

(b) If $\propto, \beta, \Gamma, \delta$ be the roots of equation.

$x^4 + px^3 + qx^2 + rx + s = 0$ then find value of

(i) $\sum \propto^2 \beta \Gamma$ (ii) $\sum \propto^3$

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