V Semester B.A./B.Sc. Examination, October/November 2012
(Semester Scheme)
MATHEMATICS – V

Time : 3 Hours Max. Marks : 90

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** questions. **(15x2=30)**

1) In a ring (R, +, .) prove that $a.0 = 0 = 0.a \ \forall a \in R$ where '0' is the additive identity of R.

2) Define a subring. Give an example.

3) Define left ideal and right ideal of a ring.

4) Show that $3z = \{3n / n \in z\}$ is an ideal of the ring (z, +, .).

5) If $f : R \to R'$ where R and R' are two rings is a homomorphism, then prove that $f(0) = 0'$.

6) If R/I is a ring of residue classes of I in R prove that if R has unit element 1 so also has R/I, a unit element I + 1.

7) Define a) Curvature b) Torsion at any point on a space curve.

8) Show that $\frac{d}{dt}\left[\bar{f} \times \frac{d\bar{f}}{dt}\right] = \bar{f} \times \frac{d^2\bar{f}}{dt^2}$.

9) Find the unit tangent vector for the curve $x = t, y = t^2, z = t^3$ at t = 1.

10) Show that the necessary and sufficient condition for a curve in space to be a straight line is that the curvature k = 0 at all its points.

11) Find the cartesian co-ordinates of the point whose cylindrical co-ordinates are (2, 60°,1).

12) If $\phi(x, y, z) = x^2y^2z^2$ find $\nabla\phi$.

P.T.O.

13) Find 'a' so that the vector.

$\vec{F} = (3x + 3y + 4z)\hat{i} + (x - ay + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal.

14) Show that Curl (grad ϕ)=0.

15) If $\phi = x^2 - y^2 + 4z$ show that $\nabla^2 \phi = 0$.

16) If $\bar{a}$ is a constant vector, find Curl $(\bar{r} \times \bar{a})$.

17) Show that $1 + x - x^2 = \frac{2}{3}P_0(x) + P_1(x) - \frac{2}{3}P_2(x)$.

18) Show that $P_n'(1) = \frac{n(n+1)}{2}$.

19) Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

20) Prove that $\int_a^b J_0 J_1 dx = \frac{1}{2}\left[J_0^2(a) - J_0^2(b)\right]$.

II. Answer **any four** of the following questions : **(4×5=20)**

1) Prove that R = {0,1,2,3,4} is a ring under addition modulo 5 and multiplication modulo 5.

2) If R is a ring such that $a^2 = a \ \forall a \in R$.

Prove that :

i) $a + a = 0 \ \forall a \in R$

ii) $a + b = 0 \Rightarrow a = b$

iii) R is commutative.

3) If p is an integer, then pz is a maximal ideal of (z, +, .) if and only if p is prime.

4) Find all the principal ideals of $R = \{0, 1, 2, 3, 4, 5, 6, 7\}$ with respect to addition modulo 8 and multiplication modulo 8.

5) If $\phi : R \to R'$ is an isomorphism of ring then isomorphic image of a field is a field.

6) An ideal K of a commutative ring R with unity is maximal if and only if the residue class R/K is a field.

III. Answer **any three** of the following : **(3×5=15)**

1) For the space curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$ find the unit tangent $\hat{t}$ and the principal normal $\hat{n}$.

2) Derive Serret-Frenet formulae for a space curve.

3) Find the equation of the osculating plane at $t = 0$ for the curve $x = 3\cos t$, $y = 3\sin t$, $z = 4t$.

4) Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at $(1, -2, 1)$.

5) Express the vector $\vec{F}$ in cylindrical co-ordinates where $\vec{F} = z\hat{i} - 2x\hat{j} + y\hat{k}$.

IV. Answer **any three** of the following questions : **(3×5=15)**

1) Find div $\vec{F}$ and curl $\vec{F}$ given $\vec{F} = \text{grad}\,(x^2 + y^2 + z^2)$.

2) Prove that $\nabla^2(r^{n+1}) = (n+1)(n+2)r^{n-1}$ where $|\vec{r}| = r$.

3) Show that $\nabla.\left\{\frac{f(r)}{r}\vec{r}\right\} = \frac{1}{r^2}\frac{d}{dr}\left\{r^2 f(r)\right\}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

4) Show that $\vec{F} = (\sin y + z\cos x)\,\hat{i} + (x\cos y + \sin z)\hat{j} + (y\cos z + \sin x)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

5) For any scalar field ϕ and any vector field $\vec{F}$ prove that Curl $(\phi\vec{F}) = \phi$ curl $\vec{F} + (\text{grad}\,\phi) \times \vec{F}$.

V. Answer **any two** of the following : **(2×5=10)**

1) State and prove Rodrigue's formula.

2) Prove that $P_n'(x) = xP_{n-1}'(x) + nP_{n-1}(x)$.

3) Evaluate $\int_{-1}^{1} x^3 P_4(x)dx$.

OR

Prove that $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$.

4) Prove that

$$\cos(x\sin\theta) = J_0(x) + 2\sum_{1}^{\infty} J_{2n}(x)\cos 2n\theta.$$