AS – 221

VI Semester B.A./B.Sc. Examination, April/May 2012
(Semester Scheme)
MATHEMATICS (Paper – VIII)

Time : 3 Hours Max. Marks : 90

***Instruction** : Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Find the locus of the point z, satisfying $|z-1| \geq 2$.

2) Evaluate $\lim_{z \to e^{\frac{i\pi}{4}}} \left(\frac{z^2}{z^4 + z^2 + 1} \right)$.

3) Prove that $f(z) = \bar{z}$ is not analytic.

4) Show that $u = x^3 - 3xy^2$ is a harmonic function.

5) Define cross ratio of four points.

6) Find the invariant points of the bilinear transformation of $W = \frac{i-z}{z+i}$.

7) Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along the path y = x.

8) State generalised Cauchy's integral formula.

9) Evaluate $\int \frac{e^z}{z} dz$ where c is the unit circle with centre at origin.

10) Evaluate $\oint \frac{\sin 3z}{\left(2 + \frac{\pi}{2}\right)} dz$ where c : | z | = 3.

11) Define :

i) Complex Fourier transform.

ii) Inverse complex Fourier transform.

P.T.O.

12) Prove that $F\{e^{iat} f(t)\} = \hat{f}(s+a)$.

13) Assuming $F(\cos x^2) = \frac{1}{\sqrt{2}} \cos\left(\frac{\alpha^2}{4} - \frac{\pi}{4}\right)$ find $F[x \cos x^2]$.

14) Using shift property $F_s\{f(x) \cos ax\} = \frac{1}{2}\left[\hat{f}(s+a) + \hat{f}(s-a)\right]$. Find the Fourier sine transform of $\frac{\cos x}{\sqrt{x}}$.

15) Given $F_c\{e^{-ax}\} = \sqrt{\frac{2}{\pi}} \frac{a}{a^2+s^2}$ find $F_s[e^{-ax}]$.

16) Write the general formula for Secant method for solving $f(x) = 0$.

17) Using bisection method, find a real root of $f(x) = x^3 - 3x - 5 = 0$ between 2 and 3 in two steps.

18) Use power method to find the largest eigen value of the matrix $A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$. Give two iterations only.

19) Using Euler's method, solve $\frac{dy}{dx} = x + y$ with the initial value $y(0) = 1$ for $x = 0.1$ in two steps.

20) Write the Runge-Kutta method to solve $\frac{dy}{dx} = f(x, y)$ with initial conditions $x = x_0$, $y = y_0$.

II. Answer **any four** of the following : **(4×5=20)**

1) Show that $\text{Amp}\left(\frac{z-1}{z+1}\right) = k$ represents a circle.
2) Derive the necessary conditions for a function $f(z) = u + iv$ to be analytic.
3) If $f(z) = u + iv$ and $u - v = (x - y)(x^2 + 4x + y^2)$ find $f(z)$.
4) Prove that a bilinear transformation preserves the cross ratio of four points.
5) Find the bilinear transformation which maps $z = \infty, 1, 0$ into $w = -1, -i, 1$.
6) Discuss the transformation $w = e^z$.

III. Answer **any two** of the following : **(2×5=10)**

1) Evaluate $\int (x^2 - iy^2)dz$ along $y = 2x^2$ from (1, 2) to (2, 8).
2) State the prove Cauchy's integral formula.
3) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ where $C : |z| = \frac{3}{2}$.
4) Prove that every polynomial equation of degree $n \geq 1$ with real or complex coefficients has at least one root.

IV. Answer **any three** of the following : **(3×5=15)**

1) Express $f(x) = \begin{cases} 1 \text{ for } |x| \leq 1 \\ 0 \text{ for } |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.
2) Find the Fourier transform of $f(x) = x\, e^{-|x|}$.
3) Find the Fourier cosine transform of the function

$$f(x) = \begin{cases} 1 + x, \text{ for } 0 < x < 1 \\ 0 \quad , \text{ for } x > 1 \end{cases}$$

4) Find the inverse sine transform of $F_s(\alpha) = \frac{e^{-a\alpha}}{\alpha} (a > 0)$.
5) Given $F_c[e^{-x^2}] = \frac{1}{\sqrt{2}} e^{-\frac{\alpha^2}{4}}$, find $F_c\, [x^2\, e^{-x^2}]$.

V. Answer **any three** of the following : **(3×5=15)**

1) Use Regula-Falsi method to find a real root of the equation $x^3 - 4x - 9 = 0$ between 2 and 3.
2) Using Newton-Raphson method, solve $x^3 - x^2 - x - 3 = 0$ in (2, 3) correct to 3 places of decimals.
3) Solve by Gauss-Jacobi method upto fourth iteration.

   $10x + 2y + z = 9,\ x + 10y - z = -22,\ -2x + 3y + 10z = 22.$
4) Using power method to find the largest eigen value of the matrix

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

5) Using Taylor's series method to find y at x = 0.1, 0.2 considering terms upto the third degree given $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 0$.