# B.Sc. I – Mathematics (Second Paper), 2006
## (Calculus)

**Note :** Attempt questions in all sections.

### Section A

**Inst. :** Attempt all questions.

**1.** Define $\varepsilon - \delta$ definition of limit of a function.

**2.** The radius of curvature at the point (2, 2) for the curve $xy = 4$ is

(a) $\sqrt{2}$ (b) 2 (c) $\frac{1}{\sqrt{2}}$ (d) $2\sqrt{2}$

**3.** Value of $\theta^n \log(1 + x)$

(a) $\frac{(-1)^n n!}{(1+x)^n}$ (b) $\frac{(-1)^{n-1}(n-1)!}{(1+x)n}$

(c) $\frac{(-1)^n (n-1)!}{(1+x)^1}$ (d) $\frac{(-1)^n (n-1)!}{(1+x)^n}$

**4.** If $f(x)$ is an even function then $\int_{-a}^{a} f(x)\,dx =$

(a) 0 (b) $2\int_0^a f(x)\,dx$ (c) $\int_{-a}^{0} f(x)\,dx$ (d) None of these

**5.** The order of the differential equation

$$\frac{d^5y}{dx^5} - 4\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0 \text{ is :}$$

(a) 5 (b) 4 (c) 3 (d) 7

**6.** Which of the following equation is Clairaut's equation :

(a) $x = py + f(p)$ (b) $y = px + f(c)$

(c) $y = px + f(p)$ (d) $y = x^2 + c$

**7.** The asymptotes parallel to y-axis of the curve $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$ is ...

**8.** Curve $x^3 + y^3 = 3axy$ symmetrical about .....

**9.** The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(a) $ab$ (b) $\frac{\pi}{4}ab$ (c) $\frac{\pi}{2}ab$ (d) $\pi ab$

**10.** State Taylor's theorem.

## Section B

**Inst. :** Attempt any eight questions.

**11.** By using $\varepsilon-\delta$ method, prove that :

$$\lim_{x \to 3} (x^2 + 2x) = 15$$

**12.** State and prove Maclaurin's theorem.

**13.** If $y = a \cos(\log x) + b \sin(\log x)$, then prove that $x^2 y_2 + x y_1 + y = 0$

**14.** If $y = 4x^2$, then find the tangent of the curve at (1, 1)

**15.** Find the asymptotes of the following curve

$$x^3 + 2x^2 y - xy^2 - 2y^3 + xy - y^2 = 1$$

**16.** Solve the differential equation :

$$y dx + (1 + x^2) \tan^{-1} x \, dy = 0$$

**17.** Solve the differential equation :

$$x \frac{dy}{dx} + y = y^2 \log x$$

**18.** Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

**19.** Find the whole area of the circle $x^2 + y^2 = a^2$.

**20.** Trace the curve $y^2 = x^3$

**21.** Prove that the derivative of even function is odd function.

## Section C

**Inst. :** Attempt any two questions.

**22.** If $\rho_1$ and $\rho_2$ be the radii of curvature at the extremities of two conjugate diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that

$$(\rho_1^{2/3} + \rho_2^{2/3}) a^{2/3} b^{2/3} = (a^2 + b^2)$$

**23.** Solve :

$$x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + 13y = \log x$$

**24.** Show that whole length of the curve :

$$x^2 (a^2 - x^2) = 8a^2 y^2 \text{ is } \pi a \sqrt{2}$$

**25.** If $\int_0^{\pi/4} \tan^n x \, dx$, then show that $\phi(n) + \phi(n-2) = \frac{1}{n-1}$ and deduce the value of $\phi(5)$.