M.G.K.V.P. University, Varanasi - 2017
Mathematics - I (BCA 110)

Note : Attempt any **five** questions. **All** questions carry equal marks.

1. (a) For the four sets A, B, C and D, prove that **8**
$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

   (b) 900 students appeared for two papers in Mathematics, 740 students passed in paper I and 660 passed in paper II. If 640 students passed in both, find the number of students who failed in both. **7**

2. (a) Prove that the relation R defined on the set of positive integers $(x, y) \in R$ if $x - y$ divisible by 5 is an equivalence relation. **7**

   (b) If $f: A \to B$ and $g: B \to C$ be one-to-one onto mappings, prove that gof is also one-to-one onto and $(g \circ f)^{-1}$ f–10g^{-1}. **8**

3. (a) A relation R on the set Z of integers is defined as follows : **7**
$m\ R\ n \Leftrightarrow m + n$ is even all m, *and* $n \in z$. *Is* R a partial order relation? Prove a give a counter example.

   (b) Let $A = \{1, 2, 3, 4\}$ and consider the relation. $R = [(1, 1), (2, 1), (2, 2), (3, 1), (3, 1), (3, 3), (3, 4), (4, 4)\}$.
Show that R is a partial ordering and draw its Hasse diagram. **8**

4. (a) Consider the subsets {2, 3}, {4, 6} and {3, 6} in the poset {(1, 2, 3, 4, 5, 6), 1}.Find for each subset if exists. **8**
   (i) Upper and lower bound,
   (ii) Greatest lower bound and least upper bound.

   (b) In a distributive lattice if an element has a complement, then prove that this complement is unuque. **7**

5. (a) If $z = x^2 \tan^{-1}(y/x) - y^2 \tan^{-1}(x/y)$, prove that: $\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{x^2 - y^2}{x^2 + y^2}$.

   (b) Divide 24 into three parts such that the continued product of the first, the square of the second and the cube of the third may be a maximum. **8**

6. (a) Find the equation of the plane which contains the line of intersection of the planes $x + y + z = 3$ and $2x - y + 3z = 4$ and parallel to the line joining the points (2, 1, 1) and (3, 2, 4). **7**

   (b) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as a great circle. **8**

7. (a) Evaluate $\iint (x^2 + y^2)\ X\ dn\ dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. **8**

   (b) Find the volume of the region bounded by the surface
$y = x^2$; $x = y^2$; $z = 0$ and $z = 3$.