M.G.K.V.P. University, Varanasi - 2018

Mathematics - I (BCA 110)

Note : Attempt any five questions. All questions carry equal marks.

Note : The answer to short questions should not exceed 200 words and the answers to long questions should not exceed 500 words.

1. (a) For the sets A, B, C show that — 7½

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $(A \cup B)' = A' \cap B'$

(b) Let $A = \{1, 2, 4\}$, $B = \{2, 5, 7\}$ and $C = \{1, 3, 7\}$, show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. — 7½

2. (a) Show that the relation R on the set z of integers defined as $R = \{x, y) : (x - y)$ is an integer$\}$ is an equivalent relation. — 7

(b) (i) Prove that the function $f : R \to R$ given by $f(x) = |x|$, $x \in R$ is neither one-one for onto. — 4

(ii) If $f : R$ and $\to R$ $g : R \to R$ be the functions defined by $f(x) = x^2 + 1$ and $g(x) \sin x$, then find gof and fog. — 4

3. (a) Define partial order relation on a non empty set A.

Show that (A, ⊆) where A is a collection of all subsets of a given set X and ⊆ is set inclusion relation on A, is a POSET. — 8

(b) Let X be the set of factors of 12 and let '≤' be the relation 'devides' i.e. $x \leq y$ iff x/y, $x, y \in X$ then draw the Hasse diagram of $(x, \leq)$. — 7

4. (a) Prove that any chain is a lattice. — 7½

(b) Define complemented lattice and sublattice. Prove that the interval $[a, b]$, where $a \leq b$ and a, b are elements of lattice L, is a sublattice. — 7½

5. (a) If $z = e^{x^2+y}$, then find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$. — 7

(b) Find the maxima and minima of the function — 8

$f(x, y) = x^3 + y^3 - 3x - 12y + 20$

6. (a) Find the equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and which is parallel to the x-axis. — 7

(b) Find the shortest distance between the lines — 8

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

and $$\frac{x-3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

7. (a) Evaluate — 7

$$\iint \frac{x-y}{x+y}\, dx\, dy \text{ over } R = [0, 1 ; 0, 1]$$

(b) Evaluate — 7

$I = \iiint xyz \, dx dy dz$ over the domain bounded by $x = 0, y = 0, z = 0, x + y + z = 1$